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2005 is physics year

2005 marks the centenary of the publication by Albert Einstein of what became one of the most celebrated texts in the history of science: "Zur elektrodynamik bewegter Körper" (On the electrodynamics of moving bodies). This 30-page paper, published in 1905 in the "Annalen der Physik", lays the foundations of the theory of special relativity. It was one among several other papers submitted the same year, making 1905 Einstein's *annus mirabilis*. Amongst the other papers published by Einstein the same year is a short article titled "Ist die Trägheit eines Körpers von seinem Energieeinhalt abhängig?" (Is the inertia of a body dependent on its energy content?). In this paper is derived a relation that has become familiar to everybody under the form ***E = mc2***. It asserts the equivalence of energy and matter and, in particular, states that matter can be transformed into energy. This possibility found its first application forty years later with the fission of heavy nuclei, unfortunately in an atom bomb. The very forces that had been unleashed for destruction however could also be harnessed for peaceful uses. This was commemorated in 2003, including by this web site, with the fiftieth anniversary of the "Atoms for Peace" initiative launched by President Eisenhower. The objective was to put the same equation to work in controlled fission chain reactions to generate electricity. In the coming decades of this century, the same equation might once again preside over the success of fusion and so provide an inexhaustible source of energy for mankind[1].

While concentrating here on Einstein's famous papers, we think it fair to also acknowledge the contributions of several other distinguished scientists such as Hendrik Lorentz, Jules-Henri Poincaré and Max Planck. They greatly contributed to the extraordinary development of the physical sciences, including relativity theory, at that time. (This observation should not be construed however as casting doubt on the value of Einstein's overall contribution to science; click here for more on this topic).

To participate in this year's celebrations and for the reasons explained above, the European Nuclear Society wishes to focus on

$$E = mc^2 ,$$

the most famous equation of all times. It has become part of everybody's culture, but few remember what is behind it. We have therefore gathered here three proofs for the benefit of those who would like to fill this gap. The arguments have been kept as short as possible. They assume however that the reader is acquainted with the principles of classical mechanics and the basics of special relativity.

The standard derivation

The derivation of $E = mc^2$ usually found in modern textbooks is based on the following formula, obtained when applying the theory of special relativity to

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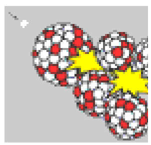

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the dynamics of a particle:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the mass of the particle when it is at rest

c is the speed of light

v is the speed of the particle

E is the total energy of the particle at speed v.

One readily observes that when the speed of the particle is set to zero, its energy does not vanish. It takes a rest value E_0 that is precisely equal to m_0c^2. The difference E – E_0 is equal to

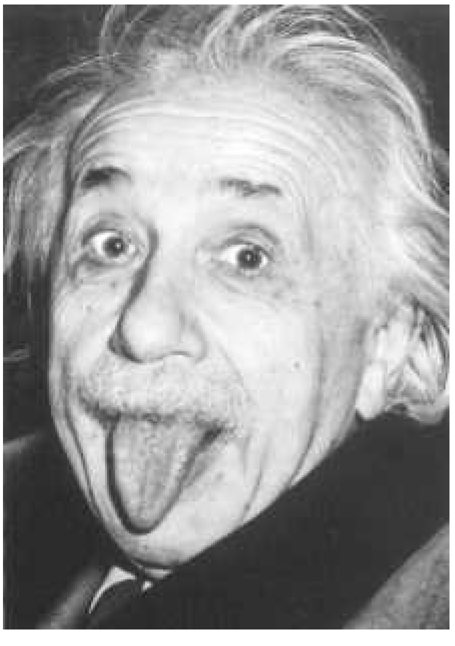
$$m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right) \quad (1)$$

which reduces to $\frac{1}{2} m_0 v^2$ for values of *v* that are very small compared to *c*. Since $\frac{1}{2} m_0 v^2$ is the kinetic energy *K* (in the classical sense of the term) of a particle of mass m_0 moving at speed *v* , one can write:

$$\mathbf{E = E_0 + K}$$

The total energy E of the particle is the sum of its rest energy E_0 and of its kinetic energy K. E_0 so emerges as the energy the particle possesses simply as a result of having a rest mass m_0.

Einstein's original derivation



The above derivation is not the one initially presented by Albert Einstein. His original derivation, as outlined in his 1905 paper, is summarised below.
In the paper titled "*Is the inertia of a body dependent on its energy content?*", Albert Einstein takes as starting point a formula established in his main paper on special relativity (On the electrodynamics of moving bodies).

Let E be the energy of a system of plane light waves measured in a co-ordinate system in which the light source is at rest. The light rays are emitted in a direction making an angle φ with the x-axis. The said formula gives the energy E* of the same light source when measured in a co-ordinate system moving at uniform speed v along the x-axis of the "rest" co-ordinate system:

$$E^* = E\frac{1-\frac{v\cos\varphi}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

(The notations have been adapted: in particular, in the original paper, the symbol for the speed of light was V)

A. Einstein then proceeds with the following thought experiment. He considers a body characterised by a quantity of energy E_0 in the rest co-ordinate system and by a quantity of energy H_0 in the moving co-ordinate system. This body starts now emitting a plane light wave of energy *L/2* in a direction making an angle φ with the x-axis and simultaneously another light wave of equal energy in the opposite direction. This body remains at rest in the rest co-ordinate system. Albert Einstein calls the energy of the body after the light emission respectively E_1 and H_1. The principle of relativity stipulates that the laws of physics must be the same in both co-ordinate systems since ones operates a uniform translation with respect to the other. One can therefore write:

$$E_0 = E_1 + (\frac{L}{2}+\frac{L}{2}) \textit{ and } H_0 = H_1 + \left(\frac{L}{2}\frac{1-\frac{v}{c}\cos\varphi}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{L}{2}\frac{1+\frac{v}{c}\cos\varphi}{\sqrt{1-\frac{v^2}{c^2}}}\right) = H_1 + \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

By subtracting these two equalities, one obtains:

$$(H_0 - E_0) - (H_1 - E_1) = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \quad (2)$$

Einstein then notes that the quantities *E* and H represent the energy of the same body expressed in two co-ordinate systems in relative motion with respect to each other, one of these being the co-ordinate system in which the body is at rest. The difference H – E can therefore differentiate itself from the kinetic energy of the body (with respect to the co-ordinate system in which it is moving) only by an additive constant C. This constant depends only on the arbitrary additive constants used to define E and H. One can therefore write:

$$H_0 - E_0 = K_0 + C, \;\; H_1 - E_1 = K_1 + C$$

which then leads to rewriting (2) as

$$K_0 - K_1 = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

The expression between brackets, already encountered above in equation (1), reduces to $\frac{1}{2}v^2$ when v is small with respect to c. The said expression becomes in such case:

$$K_0 - K_1 = \frac{L}{c^2}\frac{v^2}{2}$$

From which it follows immediately that, should a body emit a quantity of energy ***L*** in the form of radiation, its mass will be reduced by a quantity ***L/c²***. Einstein concludes his note by stating that the actual type of energy emitted is unimportant and that his theory could be tested by measuring the change of mass of substances for which the mass-energy conversion rate is high, e.g. radium salts.

Note of the editor: Einstein's famous equation does not appear explicitly in his paper. Furthermore, if it had been made explicit, the context of its derivation would have naturally led the author to write it under the form

$$\Delta m = \frac{\Delta E}{c^2}$$

Such formulation is preferable to the one under which it entered history. Contrary to the latter, the former shows clearly that

- only variations of mass and energy are to be considered;
- it is the finiteness of the speed of light c that is responsible for the equivalence between mass and energy. Should c be infinite, as assumed in Newtonian dynamics, any change in energy would result in a zero change of mass. The formula **$E = mc^2$** is not as clear in this respect, since it could lead one to think that E would become infinite for an infinite c.

A non relativistic derivation by A. Einstein

That **$E = mc^2$** can be proved without having recourse to the theory of the relativity is perhaps not so well known. Albert Einstein did provide such a derivation based on the fact that radiation exerts a pressure. The simplest way of demonstrating this fact would be to use the relation linking the energy E of a particle of mass m_0 to its momentum **p** = m**v**:

$$E^2 - p^2c^2 = m_0^2c^4 \quad (3)$$

It would then suffice to note that the mass of light being equal to zero, the above equation yields $p = E/c$ when m_0 is set to zero, which is the relation we need as starting point. The momentum *p* is here the momentum transferred to an absorbing surface by a short flash of light; it is equal to *E/c*, where *E* is the energy of the light flash. As noted by physicist Max Born, another Nobel Prize winner, "Exactly the same pressure is experienced by a body which emits light, just as a gun experience a recoil when a shot is fired".

But this derivation of $p = E/c$ will not do in the present context! Equation (3) also yields $E = m_0c^2$ when one sets p to zero, as would be the case of a particle at rest. Obviously, we cannot use as starting point a relation that already implies what we want to demonstrate. This is why it is absolutely indispensable to use in the present case the demonstration based on Maxwell's electromagnetics theory of the electromagnetic field. It must be added that this demonstration was confirmed experimentally as early as 1890 (see Max Born for further details – reference given below).

We reproduce now the announced derivation of $E = m_0c^2$, as recounted by **Max Born** in his book **Einstein's Theory of Relativity** (Dover Publications, Inc., New York, 1962). The text below is found in Chapter VI: Einstein's Special Principle of Relativity, Section 8: The Inertia of Energy (pages 283-286).

[Let us] "imagine a long tube at whose ends are two bodies **A** and **B** which are exactly equal and are composed of the same material and which, according to ordinary ideas, have the same mass (Fig. 1). But the body **A** is to have an excess of energy **E** over that of **B**, say in the form of heat, and there is to be an arrangement (concave mirror or something similar) by which this energy **E** can be sent in the form of radiation to **B**. Let the spatial extent of this flash of light be small compared with the length **l** of the tube (Fig. 1).

Then **A** experiences the recoil **E/c**. If this were transferred to the whole tube of mass **M** this would acquire a velocity **v** given by the momentum equation

$$Mv = E/c.$$

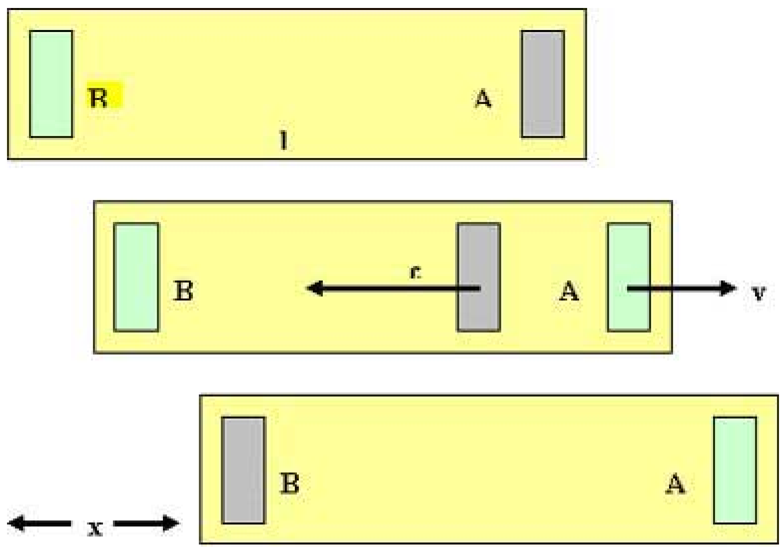


Fig. 1 A tube with two equal bodies, **A** and **B**, at its ends. **A** carries an energy **E** which is sent from **A** to **B** in the form of a light flash with velocity **c**; the recoil produces a velocity **v** of the tube. When **E** is absorbed by **B**, the tube is at rest again, but displaced by a distance **x**.

Now the transfer of momentum to the tube does not happen instantaneously; for if the tube were rigid the forces would propagate with a velocity larger than that of light. In fact, the propagation of the recoil through the tube from **A** to **B** is due to the elastic forces in the wall of the tube which are much slower than light. One has therefore to regard the process as consisting of two separate parts: (1) the emission from **A**, and (2) the absorption at **B**, and then to consider their effect on the tube, independent of one another, at a moment of time so late that not only has the elastic movement excited by the impacts expanded over the whole tube but also all elastic vibrations have died out and only the displacements of the whole tube are left over. In order to obtain the total effect one has to add the two displacements due to the impacts at **A** and **B** because elastic waves (of small amplitude) superpose undistortedly.

1) The recoil at **A** transfers a movement to the tube in such a way that the late instant t_1 when all vibrations have disappeared its velocity is **v** and its displacement

$$x_1 = vt_1$$

2) When the light is absorbed at **B** the tube receives a movement of which at

the instant $\mathbf{t_1}$ only a resultant velocity in the opposite direction **–v**, is left over; the corresponding displacement is

$$\mathbf{x_2 = -v(t_1 - t)},$$

if **t** is the time light needs to travel from **A** to **B**; for the impact on **B** happens the time interval **t** later. The sum of the two displacements is

$$\mathbf{x = x_1 + x_2 = vt},$$

the same as if the tube were rigid[2]. If we substitute here $\mathbf{v} = \frac{\mathbf{E}}{\mathbf{Mc}}$ and $\mathbf{t} = \frac{\mathbf{l}}{\mathbf{c}}$ we obtain for the displacement of the tube

$$\mathbf{x} = \frac{\mathbf{El}}{\mathbf{Mc^2}}.$$

Now the bodies **A** and **B** may be exchanged (this may be done without using external influences). Let us suppose that two men are situated in the tube, who put **A** in the place of **B**, and **B** in the place of **A**, and then themselves return to the original positions. According to ordinary mechanics the tube as a whole must suffer no displacement, for changes of position can be effected only by external forces.

If this exchange were to be carried out, everything in the interior of the tube would be as at the beginning, the energy **E** would again be at the same place as before, and the distribution of mass would be exactly the same. But the whole tube would be displaced a distance **x** with respect to its initial position by the light impulse. This, of course, contradicts all the fundamental canons of mechanics. We could repeat the process and thus impart any arbitrary change of position to the system without applying external forces. This is, however, an impossibility. The only escape from the difficulty is to assume that when the bodies **A** and **B** are exchanged, these two bodies are not mechanically equivalent but that **B** has a mass greater by **m** than **A** in consequence of its excess of energy **E**. Then the symmetry during the exchange is not maintained, and the mass **m** is displaced from right to left by a distance **l**. At the same time the whole tube is displaced a distance **x** in the reverse direction. This distance is determined by the circumstance that the process occurs without the intervention of external influences. The total momentum, consisting of that of the tube **M x / t** and that of the transported mass **–m l / t**, is thus zero. Then

$$\mathbf{Mx - ml = 0},$$

from which if follows that

$$\mathbf{x = ml / M}.$$

Now this displacement must exactly counterbalance that produced by the light impulse, hence we must have

$$\mathbf{x} = \frac{\mathbf{ml}}{\mathbf{M}} = \frac{\mathbf{El}}{\mathbf{Mc^2}}.$$

This allow us to calculate **m** and we get

$$\mathbf{m} = \frac{\mathbf{E}}{\mathbf{c^2}}.$$

This is the amount of inertial mass that must be ascribed to the energy **E** in order that the principle of mechanics which states that no changes of position can occur without the action of external forces remains valid.

Since every form of energy is finally transformable into radiation by some process or other, this law must be universally valid. Thus we have a great unification in our knowledge of the material world: *Matter is the widest meaning of the word* (including light and other forms of pure energy, in the language of classical physics) *has two fundamental qualities: inertia, measured by its mass, and the capability of performing work, measured by its energy*. These two are strictly proportional to one another. Wherever electric and magnetic fields or other effects lead to intense accumulations of energy, they are accompanied by inertia. Electrons and atoms are examples of enormous concentrations of energy." (end of quotation)

Derivation of p = E/c based on Maxwell's theory – The pressure of radiation

Let us consider a particle of charge ***q***, initially at rest, submitted to the electromagnetic field of travelling plane waves moving along the z-axis. These waves are characterised by an electric field **E** aligned with the x-axis and a magnetic field **B** aligned with the y-axis. Furthermore, it can be shown that, for plane waves and in the MKSA system of units,

$$|E| = c|B| \text{ (a).}$$

The said particle will be subjected to the Lorentz force

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$

If we call $\mathbf{e_x}$, $\mathbf{e_y}$, $\mathbf{e_z}$ the unit vectors along the three axes of reference, we have:

$\mathbf{E} = E_x\mathbf{e_x}$, $\mathbf{B} = B_y\mathbf{e_y}$, $\mathbf{v} = v_x\mathbf{e_x} + v_y\mathbf{e_y} + v_z\mathbf{e_z}$ and (a) becomes

$$E_x = cB_y$$

while the expression of the Lorentz force becomes:

$$\mathbf{F} = q(E_x\mathbf{e}_x + v_xB_y\mathbf{e}_z - v_zB_y\mathbf{e}_x).$$

Let us now compute the average $\langle \mathbf{F} \rangle$ of force **F** over one cycle. Since the field **E** is a sine function of time, its average over one cycle is equal to zero. Now we note that during the first cycles, the particle has not had time to gather much speed in the z direction. We can therefore assume that $\mathbf{v_z}$ is almost constant, in which case the average value of the third term is proportional to the average of B_y. But B_y is also a sine function of time and its average over one cycle is also equal to zero. The average of force **F** is therefore equal to the average of the second term in the expression (b). Furthermore, according to Newton's third law, **F** is equal to d**p**/dt the rate of change of momentum of the body on which **F** is applied. One can therefore write:

$$\langle d\mathbf{p}/dt \rangle = q\langle v_xB_y \rangle \mathbf{e}_z$$

We now compute the average $\langle dW/dt \rangle$ of the work W done unit of time by the wave on the particle. Since dW/dt = **F.v**, we have $dW/dt = q\,\boldsymbol{v}.(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$ which reduces to $dW/dt = qv_xE_x$ and, since $E_x = c\,B_y$, we also have

$$\langle dW/dt \rangle = qc\langle v_x B_y \rangle$$

If we compare the last two formulas, we see immediately that

$$\langle d\mathbf{p}/dt \rangle = \mathbf{e}_z \langle dW/dt \rangle / c$$

This last expression indicates that, in a period of time during which the plane waves impart a quantity of energy ΔW to the particle, they also impart it a increase of momentum Δp equal to $\Delta W/c$.

In defence of Einstein

This year's celebrations have unfortunately given rise to a peripheral controversy regarding Albert Einstein's actual merit as a scientist. It is a regrettable feature of our times that there will always some people who find it appropriate to try and spoil celebrations and make us believe that established values are unfounded. Building on the fact that Einstein did not give any credit to his predecessors in the above-mentioned 1905 papers, they have thought it fit to push the argument further. They have painted Einstein as a pla-gia-rist[3] who was not the first to propose the famous mass energy relationship, who derived it incorrectly and even who was not the actual author of the theory of general relativity.

It is true that Einstein did not quote any reference in his 1905 papers. But even a cursory review of his scientific career makes it abundantly clear that he made first class contributions to physics. The anteriority question was taken due care of when he was awarded the Nobel Prize in Physics 1921 for "his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect" and not, as one would have expected, for the relativity theory. Albert Einstein was a respected member of the scientific community from 1905 until his death in 1955. It is essential to observe at this stage that those lambasting Einstein today have no more supporting evidence for doing so than was available 50 or 80 years ago. If anything, there is less evidence available today since all the protagonists have long passed away. Unless one assumes that it is possible to fool everybody during fifty years, one must come to the logical conclusion that Albert Einstein' recognition by his peers was not unfounded. Only speculation of dubious scientific value coming a century after the facts can contradict the first-hand experience of Einstein's contemporaries. To make this last statement clear, one need only observe that it would be equally easy to make a similar case against Isaac Newton:

- he did not find out the inverse square relationship governing attraction force (it was his colleague Robert Hooke, or at least the latter claimed he did),
- one can claim that he was not the first to invent differential calculus and give Leibniz priority in this field,
- one could claim that he was not entitled to carry out the operations relating to his "fluxion" calculus (bishop Berkeley actually did object and the matter was not resolved until the middle of the 19th century thanks to Weierstrass) and, to cap it all,
- he was solitary, suspicious and bad tempered.

This single example should make it clear that the type of malicious criticism this year's celebrations have unfortunately prompted reflects more on the mediocrity of their authors than on the individuals they are trying to disparage. Furthermore, they insult Einstein's contemporaries by implying that they were too stupid to understand what was going on although they had access to first-hand evidence.

1) For further details on fission and fusion as energy generating reactions, see "Binding energy" in the Glossary of nuclear terms on this web site.

2) Einstein's first derivation (1905) supposed the tube to be rigid. Later (1907) he himself criticized the concept of a rigid body in the theory of relativity. Our [Max Born's] derivation is a simplified version of a consideration by E. Feenberg.

3) hyphenation added to prevent search engines from this listing this web page on searches associating Albert Einstein with the p-word

Reference: Berkeley physics course – volume 3, chapter 7, pp 362-364 (the derivation is provided in CGS units).